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On the Suitability of Econometric Demand Models in Design for Market Systems

A goal of design for market systems research is to predict demand for differentiated products so that counterfactual experiments can be performed based on design changes. We review conventional methods and propose an additional method to evaluate the suitability of econometric demand models estimated from revealed preference data for use in product design studies. We evaluate one demand model form from literature and two newly constructed forms for new vehicle demand along existing metrics of fit and predictive validity as well as a newly developed metric of proportional substitution sensitivity. We show that a model that includes horizontally differentiated preferences for size performs better under metrics of fit and predictive validity but that no model relaxes the IIA property satisfactorily to avoid exploitation by design optimization. We conduct design studies separately, applying each demand model form assuming the automotive market is in Bertrand–Nash price equilibrium. Results illustrate that the influence of the demand model form on the optimum in terms of design variables and expected firm profit is significant. [DOI: 10.1115/1.4002941]

1 Introduction

A goal in design for market system (DMS) research is to integrate models of demand, cost, and engineering performance of products in a comprehensive design optimization framework. The firm's ability to conduct forward-looking product planning in a market system context requires not only engineering models that link product performance to design attributes but also customer decision models that appropriately link design attributes to product demand. The DMS literature has integrated such representations of customer decisions by borrowing existing discrete choice customer utility models from the social sciences (e.g., Refs. [1–3]) or by developing new versions (e.g., Refs. [4,5]). However, little research has investigated the evaluation of these forms of customer utility for use in design optimization.

We explore three functional forms of a mixed-logit utility model applied to the automotive industry. We compare these models according to traditional criteria from the social sciences, including metrics of fit and predictive validity as well as a new substitution pattern metric we deem relevant to design studies. We examine the functional form differences of the two newly estimated mixed-logit models focusing on the representation of consumer preferences for particular vehicle attributes as horizontally differentiated, vertically differentiated, or in some combination. Results indicate that one model performs better under metrics of fit and predictive validity but no model produces satisfactory substitution patterns for this application. We show that relatively small changes in the functional form of utility can result in significantly different equilibrium price, share, and profit predictions. The functional form differences in the demand model may or may not result in different optimal vehicle design variable values depending on the feasible design space.

The functional form of utility influences the structure of heterogeneity of consumer preferences. Economic theory describes two categories of preference differentiation: vertical differentiation results when consumers agree on the relative value ordering of competing products, or attributes of a product, but differ in their willingness to pay for increased quality; horizontal differentiation results when consumers disagree about the relative ordering of goods or attribute levels of a good [6]. For products with many attributes, product ordering can vary across consumers even when all consumers have monotonic preferences that differ in magnitude. We focus on differences in relative ordering of attribute levels when discussing horizontal differentiation. In a market such as the automotive one, there may exist both vertically and horizontally differentiated preferences, e.g., various grades of luxury for a full-size sedan and e.g., varying preference for vehicle classes, respectively. This paper tests the hypothesis that differences in how the functional form of utility accounts for vertical or horizontal preferences will have a significant effect on the model's ability to represent customer preferences and on the solutions of design optimization studies. We focus only on the influence of the demand specification on optimal product designs, leaving consideration of the interaction of the demand model with the cost model, technology capabilities, and market structure for future work.

The remainder of the article is structured as follows: Sec. 2 provides the background on econometric models with specific highlights to applications in the automotive market and developments in the engineering design literature. Section 3 presents the methods for evaluating a given demand model for use in a design context. Section 4 presents the two mixed-logit models for new vehicle automotive demand and discusses their performance and performance of a model from literature with respect to the presented evaluation methods including the proposed substitution pattern metric. Section 5 discusses the implementation of these models in a design study, and Sec. 6 summarizes the conclusions.

2 Background

We focus on econometric models of product demand derived from observed consumer behavior. Following the decision-based design (DBD) literature, we assume that the customers seek to maximize their expected utility, and the demand model provides a representation of the customer utility derived from purchasing a specific good. The utility U_{ij} derived by customer i from product j is a function of product attributes $\mathbf{z}$, customer characteristics $\mathbf{w}$, and details of the specific choice context $\mathbf{k}$, $U_{ij}=f(\mathbf{z}_j,\mathbf{w}_i,\mathbf{k})$. The

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Contributed by the Design Automation Committee of ASME for publication in the Journal of Mechanical Design. Manuscript received October 30, 2009; final manuscript received October 28, 2010; published online December 7, 2010. Assoc. Editor: Timothy W. Simpson.

product attributes **z** are functions of design variables **x** that engineers control directly. Thus, engineering design decisions can be explicitly linked to demand (e.g., Ref. [7])—a key element of the DMS approach. The form of the utility function influences how customers respond to changing or removing an existing product or adding an additional product to the market, which may significantly affect the results of design studies.

2.1 Econometric Demand Modeling and Applications to the Automotive Vehicle Market. Many econometric demand models have been estimated for the automotive market [8–13]. These models are often tailored for specific analyses, such as interpreting firm pricing behavior, and do not always have forms of utility or identification strategies appropriate for design studies. For example, Berry–Levinsohn–Pakes (BLP) estimated mixed-logit models to introduce a method for estimating demand in differentiated-product markets accounting for price endogeneity, first using only aggregate level demand and known population demographic distributions [14] and then using disaggregate demographic data [15]. They simultaneously recovered demand and cost parameters assuming observed prices satisfied Bertrand–Nash equilibrium. Many succeeding papers describing the automotive market have followed or made incremental improvements to the BLP methodology (e.g., Refs. 8, 9, and 13).

Several drawbacks to the utility function and estimation assumptions in the BLP models can be identified for our present purpose. Because identification of the utility parameters was done assuming design decisions were exogenous, the resulting models do not necessarily support counterfactual analyses based on changes in product attributes [16]. Additionally, the utility specification enforces counterintuitive notions about consumer preferences. For instance, the utility specification in Ref. [14] contains the attribute "miles per dollar" (fuel economy divided by the price of gas), which lowers the relative importance of fuel economy as gas price increases. More recent demand models have corrected this in favor of a "dollars/mile," "gal/mile," or mpg attribute (e.g., Refs. [9,12,15]) but in all these models utility is monotonic with respect to vehicle size, when it seems plausible that some individuals would prefer a vehicle size smaller than the largest vehicle in the market.

2.2 Econometric Model Application in Engineering Design. Engineering researchers have begun to adopt, modify, and develop econometric models to serve in DMS applications, especially applied to automotive design [1–3,17,18]. Michalek et al. [1] and Shiau and Michalek [4] extracted the price, fuel economy, and acceleration pieces of utility from an existing automotive demand model, assuming all other product attributes (e.g., dimensions) are fixed. Frischknecht and Papalambros [3] used these same assumptions while including vehicle size decisions but also adjusted the parameters for fuel economy and acceleration, recognizing the fleet average (i.e., consumers' expectations) of these attributes have changed since the time the model was estimated. These heuristic methods allow demand models to be used in design optimization for illustrative purposes but the interpretation of the results is uncertain.

Other studies (e.g., Refs. [4,5,17] have constructed new demand models for the purpose of design optimization. However, little emphasis has been placed on the choice of the functional form of utility used in these models. Wassenaar et al. [17] presented a first attempt at addressing this issue, applying the Kano method to select the functional form for each product attribute but offered no methods for evaluating the suitability of the resulting demand model for design optimization beyond measures of fit. Wassenaar and Chen [19] is an exception to this trend; these authors reported that 200 forms of utility were evaluated on in-sample and out-of-sample fit before applying the model to a design optimization study.

Markets with numerous product alternatives such as the automotive market face the additional challenge of scale. Several examples from the DMS literature [20–22] have been conducted for a single firm, and the set of competing product designs has been assumed fixed even when price competition is considered [23]. The total number of products in the market was relatively small (2–25) compared with the automotive market (200+). When estimating consumer utility for a market of this large size, evaluating a large set of alternative utility function specifications quickly becomes infeasible. There is then value in understanding the effect of broader properties of utility functions so that a smaller set of functional forms can be explored.

Evaluating demand models with additional criteria to in-sample fit is important for DMS research because we conjecture that a given model could fit existing data well but perform poorly in a design simulation context. This can occur when the design study focuses on a different set of products or individuals from what the choice model was estimated, and when the estimated choice model describes well the aggregate consumer behavior given a fixed vehicle fleet but misses preference nonlinearities and correlations between attributes, which would mislead the design study. Comparing model fit for predicting choices out-of-sample, either in individuals or product alternatives or both, is the first step. An additional step is to develop confidence that the chosen demand model reflects realistic consumer substitution patterns. The literature to date has focused on showing demand models of increasing sophistication that exhibit more intuitively appealing substitution patterns such as the mixed-logit versus the simple multinomial logit [24]. It is unclear whether these models are adequate.

In the present article, we seek to understand the influence of the treatment of differentiated preferences, implicit in the functional form of utility, on the resulting model predictions. We review existing evaluation techniques and propose a new technique for comparing the substitution patterns predicted by a given demand model. Integration of these demand models with engineering and product cost models is not part of the present study.

2.3 Preference Differentiation in the Econometric and Engineering Design Literature. Let Eq. (1) be a general form of mixed-logit utility U_{ij} where $\boldsymbol{\delta}$ are fixed coefficients for product attributes, $\boldsymbol{\beta}$ are fixed coefficients for demographic and attribute interactions, $\boldsymbol{\mu}$ are random coefficients for product attributes or product attribute and demographic interactions, and ε is a random variable representing uncertainty in a consumer's utility, assumed to be independent and identically distributed according to an extreme value type 1 distribution for all individuals [11]:

$$U_{ij} = \boldsymbol{\delta}^{\top}\mathbf{d}_j + \boldsymbol{\beta}^{\top}\mathbf{b}_{ij} + \boldsymbol{\mu}_i^{\top}\mathbf{m}_{ij} + \varepsilon_{ij} \tag{1}$$

Here, attributes (including price and nonprice product characteristics) can enter the equation in **d**, or can enter with demographic interactions in **b** or **m**, or some combination. Note that, if $\beta^{\top}$ $\mathbf{b}_{ij}$ and $\boldsymbol{\mu}_i^{\top}$ $\mathbf{m}$ are omitted from the equation (as in Ref. [1]), the utility model will be a homogenous logit one and preferences would be neither horizontally nor vertically differentiated.

If price appears in **b** or **m**, or both (as in Ref. [11]), then the model can account for vertically differentiated preferences because individuals can differ in their willingness-to-pay for improvements in utility from nonprice attributes [6]. However, if nonprice attributes monotonically affect utility for all individuals through either **b** or **m**, then horizontally differentiated preferences will not be captured except in two special cases. The first case is when a random coefficient straddles 0 so that an increase in the given attribute provides utility to some individuals and disutility to other individuals, implying that consumers either like or dislike the attribute monotonically. The second case comes from the dummy variables that indicate categorical attributes such as vehicle class or brand: when the coefficients on these dummy variables are either random or interact with demographic information, as in Ref. [9], it is possible for the preference ordering between categories to vary across the population. Besides these cases, utility functions monotonic with respect to nonprice attributes do not model horizontally differentiated preferences because they imply that consumers uniformly agree that increasing (or decreasing) the attribute is more desirable, although they may disagree on their willingness-to-pay for this increase (decrease), i.e., vertical differentiation with respect to that attribute.

Nonmonotonic utility functions with fixed coefficients are similarly unable to model horizontally distributed preferences because all individuals agree on the ideal value(s) of a particular attribute. However, nonmonotonic functions of an attribute with random coefficients or interactions with demographics can account for horizontal preferences. An example of this functional form is to dummy code or effects code various levels of an attribute and allow preferences for each level to vary among individuals, as in Refs. [20,22,25]. This approach allows the analyst to remain agnostic about the functional form of the utility with respect to the quantitative attributes until after the estimation. While nonmonotonic utility functional forms such as these are commonly used with stated choice data, they are rarely used with observed data because of the high number of levels observed for each attribute.

A less common approach is to transform a product attribute such that it does not enter into the utility function monotonically. In the demonstration study of Sec. 5, we employ an ideal-point utility formulation for vehicle size as an example of this approach. We choose vehicle size as it is the continuous attribute most likely demonstrating horizontally differentiated preference; we expect preference for acceleration and fuel economy to be monotonic although not necessarily linear. In Sec. 3, we propose model evaluation methods before proceeding with the demonstration study.

To explore whether using a model of vertical differentiation would result in different design optimization results than a model of horizontal differentiation, we experiment with functional forms of utility that have more freedom to capture horizontal differentiation if it exists. We expect consumer substitution patterns between vehicles would be different between these two forms of preference models. We also check whether the substitution patterns of either model match our expectations about substitution in the automotive market. The demonstration study shows that substitution patterns are different between the models and different optimal designs do exist, although outside the feasible vehicle design space representing a midsize crossover vehicle. We also find that substitution between similar vehicles is smaller than anticipated.

3 Evaluation Methods

Chintagunta et al. [26] proposed four criteria for evaluating choice models in their review of the economic and marketing literature regarding structural choice models such as those described in Sec. 2. They are fit, interpretability, predictive validity, and plausibility. The econometrics literature has developed and applied many statistical tests to address these criteria in the context of choice share predictions [24].

Other properties of choice models beyond in- and out-of-sample fits are important to investigate for engineering design. Here, we offer a review of standard methods from econometrics and a newly conceived method for comparing substitution patterns across demand models in the context of a changing product choice set in order to evaluate the suitability of demand models for design optimization studies.

3.1 Interpretability and Fit. We define interpretability as a qualitative assessment of how well the functional form of utility is supported by theory or beliefs of market behavior. Questions that a modeler should ask when checking for interpretability before estimation include the following: Do all components of utility have behavioral or physical significance? Does each behavioral or physical factor influence choice probabilities in a manner that is consistent with theory or belief? After estimation, the modeler should check interpretability by conducting various tests: (1) the significance of the estimated parameters with particular attention to those deemed to support theory or beliefs; (2) the signs of the estimated parameters; (3) overfitting; and (4) colinearities among attributes. We will report on items 1 and 2 in Sec. 4. Items 3 and 4 require further development and are left for future work.

Metrics to evaluate choice model fit can be directly applied from econometrics but we deemphasize their importance compared with the other evaluation criteria. Measures of fit emphasize the descriptive power of a model with respect to the same data set used to estimate the model. However, they do not indicate if the model is correctly describing the most important factors or how well the model will predict outcomes based on changes in behavior, both of which are important for design studies.

One standard measure of fit for logit models is a likelihood ratio index $\rho^2 = 1 - LL(\hat{\boldsymbol{\beta}})/LL(0)$, which measures how well the estimated model performs compared with a model where all of the parameters are zero (i.e., no model). Here, $LL(\hat{\boldsymbol{\beta}})$ is the maximized log-likelihood ratio function given a set of parameter values $\hat{\boldsymbol{\beta}}$, which can be used to compare the goodness-of-fit of two or more models if they are estimated from identical data sets and choice alternatives [24].

The maximized likelihood values can be used directly to compute a statistical measure to evaluate the hypothesis that the specified model (or a set of parameters from the specified model) is significant. This is known as the likelihood ratio test [27]. Let $L^* = \max L(\boldsymbol{\omega}) / \max L(\boldsymbol{\Omega})$, where $L(\boldsymbol{\omega})$ is the likelihood function given a model with parameters $\boldsymbol{\omega}$ and $L(\boldsymbol{\Omega})$ is the likelihood function given a model with parameters $\boldsymbol{\Omega}$, and where $\boldsymbol{\omega}$ is a subset of $\boldsymbol{\Omega}$. If the null hypothesis is true (that the model with parameters $\boldsymbol{\Omega}$ is not statistically different from the model with parameters $\boldsymbol{\omega}$), the value of $-2 \ln L^*$ has been shown for large samples to be approximately chi-square distributed with degrees of freedom M equal to the difference in the number of parameters for each model, namely, $-2 \ln L^* \sim \chi^2_M$. The value of $-2 \ln L^*$ can be compared with the critical value at a given confidence interval for a random variable distributed as χ^2_M.

3.2 Predictive Validity and Plausibility. The goal of estimating the demand model in DMS is to predict demand for products in counterfactual experiments or design scenarios in the engineering literature. Model properties of particular interest to design studies include prediction of how consumers trade-off product attributes and their willingness to pay for improving an attribute.

Whereas fit measures the ability of the model to describe the in-sample data, predictive validity evaluates the ability to describe out-of-sample data. These may include a hold-out sample from the same time period for which the model was estimated or a sample from another time period or population. The likelihood ratio index can be used to evaluate a model's ability to predict choice shares from an out-of-sample data set. Such evaluation is valuable because superior model performance (i.e., higher ρ^2) on out-of-sample data can be interpreted as capturing consumer trade-offs better rather than simply describing the data (i.e., a good fit).

We define plausibility as the ability of the estimated model to produce outcomes that represent market behavior based on theory or observations. A way to assess plausibility is to examine substitution patterns between competing goods given changes in price and other attributes, as measured by own- and cross-elasticities. The substitution patterns can then be compared with observed market behavior where possible.

The elasticity of demand $E_{jX_k^m}$ for vehicle j is the percentage change in demand for j given a percentage change in attribute X^m of vehicle k. The formulas for own- and cross-elasticities for individual i given a mixed-logit choice model are as follows [16,28]:

$$E_{ijX_k^m} = \frac{\partial P_{ij} X_k^m}{\partial X_k^m P_{ij}} = \begin{cases} \dfrac{X_k^m}{P_{ij}} \displaystyle\int B_{ik}^m L_{ik}(\boldsymbol{\mu})(1 - L_{ij}(\boldsymbol{\mu})) f(\boldsymbol{\mu}) d\boldsymbol{\mu} & \text{if } j = k \\ -\dfrac{X_k^m}{P_{ij}} \displaystyle\int B_{ik}^m L_{ik}(\boldsymbol{\mu}) L_{ij}(\boldsymbol{\mu}) f(\boldsymbol{\mu}) d\boldsymbol{\mu} & \text{if } j \neq k \end{cases} \tag{2}$$

where $P_{ij} = \int (e^{U(\boldsymbol{\mu})_{ij}} / \Sigma_K e^{U(\boldsymbol{\mu})_{ik}}) f(\boldsymbol{\mu}) d\boldsymbol{\mu}$ is the unconditional likelihood individual i chooses vehicle j from the set of all vehicles K. The definition of the vector $\boldsymbol{\mu}$ expands compared with Eq. (1) to represent the coefficients corresponding to the individual factors of the utility function including fixed and random coefficients. The function $f(\boldsymbol{\mu})$ is the joint probability distribution function of $\boldsymbol{\mu}$. $L_{ij}(\boldsymbol{\mu}_i) = e^{U(\boldsymbol{\mu}_i)_{ij}} / \Sigma_K e^{U(\boldsymbol{\mu}_i)_{ik}}$ is the conditional likelihood individual i chooses vehicle j from all vehicles K for a particular $\boldsymbol{\mu}_i$ with similar interpretation for L_{ik} for vehicle k. X_k^m is the value of attribute X^m for vehicle k and B_{ik}^m is the partial derivative of utility function U_{ik} with respect to X_k^m.

The substitution patterns inherent in a given demand model are endogenous to design optimization. Therefore, comparing the substitution patterns between vehicle designs across demand models is likely more important for interpreting design studies than comparing the elasticity of demand across attribute levels. This is because design studies involve changing the available choice set of alternatives through potentially large changes to the attribute values and the addition or removal of product alternatives. It will be difficult to interpret the results of a design study if we do not have confidence on how the demand model reapportions market shares in such circumstances.

Research has pointed to the substitution patterns shortcomings of the multinomial logit model in situations where the independence of irrelevant alternative (IIA) property is not appropriate [25,29]. More recent research has questioned the suitability of the mixed-logit model because of the substitution patterns that are implied [30–32]. The mixed-logit model allows substitution patterns across the population that allows a greater portion of the market share of a new product to be drawn from similar products. However, each individual is assumed to behave according to the IIA property. The result referenced in Refs. [30,31] and observed in this work is that, given the introduction of an identical alternative, the market share for the new entrant will be drawn substantially from all of the alternatives in the market rather than solely from the existing identical alternative or predominantly from a small group of the closest substitutes.

We adopt a substitution pattern metric Ψ from Refs. [30,31] and propose that it can be used to compare demand models for use in design studies where

$$\Psi = \frac{P_{j|\text{original}} - P_{j|\text{expanded}}}{P_{j'|\text{expanded}}} \tag{3}$$

and $P_{j|\text{original}}$ is the choice share of alternative j in the original choice set, j' is an identical alternative to j that is added to the choice set to form an expanded choice set and $P_{j|\text{expanded}}$ and $P_{j'|\text{expanded}}$ are the choice shares of alternatives j and j', respectively, in the expanded choice set.

The substitution metric informs the analyst about the degree of proportional versus perfect substitution implied by a given demand model across the feasible design space. A value of $\Psi = 1$ implies perfect substitution where alternative j' takes market share exclusively from j. A value of $\Psi = P_{j|\text{original}}$ implies proportional substitution according to the IIA property.

The procedure for computing the substitution pattern metric is as follows. Select a baseline product alternative j or set of product alternatives J with design variable values in the interior of the feasible space for design optimization. For product alternative $j = 1, \ldots, J$, introduce alternative j into the market product set and compute the estimated market share $P_{j|\text{original}}$ using the method of sample enumeration. Next, introduce a duplicate product alternative j' into the choice set and compute the market shares of the two duplicate alternatives $P_{j|\text{expanded}}$ and $P_{j'|\text{expanded}}$. Then, compute Ψ. Repeat the procedure for each alternative of interest $j = 1, \ldots, J$ and for each demand model of interest. Finally, the substitution pattern metric can be compared across demand models according to the min, max, mean, or median values as well as plots showing each of the sample points.

One drawback of Ψ is that its lower bound increases with increasing choice share for the alternative of interest. This makes it difficult to compare the substitution patterns across demand models because choice share for a particular alternative will vary across demand models, and Ψ naturally increases with increasing choice share of the alternative of interest. One possible normalization that results in a value between zero and one is to take the difference in the new alternative choice share in the augmented choice set $P_{j'|\text{expanded}}$ and what the new alternative choice share would be if treated as a perfect substitute $P_{j'^S|\text{expanded}}$, divided by the difference between what the new choice share would be according to proportional substitution $P_{j'^P|\text{expanded}}$ and what the new choice share would be if treated as a perfect substitute $P_{j'^S|\text{expanded}}$. Differencing this quantity from 1 according to Eq. (4) results in a monotonically increasing value Ψ' between 0 and 1 with respect to increasing Ψ.

$$\Psi' = 1 - \frac{P_{j|\text{expanded}} - P_{j^S|\text{expanded}}}{P_{j^P|\text{expanded}} - P_{j^S|\text{expanded}}} \tag{4}$$

The analyst must decide the substitution pattern that is desirable for a given application. For example, the appropriate substitution pattern for the automotive vehicle industry remains an open question. The substitution pattern metrics Ψ and Ψ' provide a tool to allow the analyst to compare various demand models. As the choice set size increases, each alternative takes a smaller market share. The relative difference between perfect and proportional substitution is accentuated. Therefore, differentiating between substitution patterns becomes more important in terms of the relative changes in market share. Differences in market share predictions will influence expected firm profit and thereby have the potential to influence product design decisions.

4 Vehicle Demand Modeling Example

We compare models built on different assumptions according to the methods of Sec. 3. Specifically, we compare two mixed-logit models with monotonic and ideal-point representations of vehicle size preference, respectively.

4.1 Estimation. We estimated the models according to the maximum simulated log-likelihood approach using Train's publicly available estimation code [28]. Consumer data came from the Maritz Research 2006 New Vehicle Customer Satisfaction Survey [33] and additional vehicle data came from Chrome System Inc.'s New Vehicle Database and VINMatch tool [34]. We follow the pattern of Ref. [11] in using self-reported considered vehicles to augment the purchased vehicle observation by treating the vehicles a consumer listed "also considered purchasing" as pseudo-choice-observations conditional on their purchased or higher ranked considered vehicles being removed from the choice set. Train and Winston reported success in identifying heterogeneous taste coefficients for a mixed-logit model when employing the rank-ordered logit formulation compared with little or no significant heterogeneous taste coefficients when only the single purchase observation is used for model estimation [11].

The market was represented by 473 vehicles, a subset of 2006 model year vehicle styles corresponding to available make, model, and engine options. We eliminated vehicles priced over $100,000 (a small percentage of the total market) as well as seven alternatives that were not observed in the survey data and further reduced the choice set by consolidating pickup truck and full-size van models with gross-vehicle-weight ratings over 8500 lb to two models each per manufacturer. An individual's choice sets were composed of the purchased vehicle (or stated considered vehicle) and 99 uniformly conditioned randomly selected vehicles up to a total of 100 vehicles. The choice sets for the pseudo-observations were composed of the considered vehicle and 99 uniformly conditioned randomly selected vehicles up to a total of 100 vehicles conditional on the higher ranked vehicles being excluded from the choice set. Assuming the error term ε_{ij} is independent and identically distributed extreme value type 1 in the mixed-logit model allows ranked observations to be treated as separate choice observations for a single individual [24].

The uniform-conditioning property states that a multinomial logit model estimated using choice sets composed of randomly selected members drawn with a uniform distribution from the set of all choice alternatives will result in consistent estimates of the model parameters [35]. This property holds due to the IIA property of the logit model. While the models we estimate do not exhibit the IIA property, other researchers have shown that with choice sets between 1/8–1/2 of the complete choice set, similar results are achieved compared with estimating a model with the complete choice set [36,37]. We chose to sample the choice set in our model estimations due to the computational constraints of estimating such a large model. Reducing the computational burden from larger choice sets allowed us to increase the sample size and to test many alternative specifications in the same time it would take to estimate one model with the full choice set. Additionally, we repeated the estimation procedure three times for the same sample of individuals. For each repeat estimation, new uniformly conditioned choice sets were drawn for each individual. The resulting parameter estimates were similar for each estimation.

A set of 6563 individuals was sampled from the 81,705 survey respondents that reported income and purchased a 2006 model-year vehicle using choice-based sampling to approximate 2006 market shares. In some cases, either too few vehicle choices were available in the survey to match the sales percentage or too few respondents would be sampled to represent the demographics of purchasers of a particular vehicle. To account for this, a set of weights was generated for each individual in the sample to adjust the log-likelihood calculation to correctly match 2006 market shares. We arbitrarily set a minimum of five observations for each vehicle alternative (if at least five were available) to increase the sample of demographics for consumers of low market-share vehicles. We utilized the weighting procedure available in Train's code that multiplies the log of each individual's logit probability, including all choice observations, by the weighting value for that individual. The assumption with this approach is that the sampled individuals who purchased a particular vehicle are representative of all individuals who purchased that vehicle.

We instrumented for price endogeneity ($R^2=0.78$) following Train and Winston [11], where instrumental variables consist of differences between vehicle attributes (horsepower *hp* and size, computed as the product of length *L*, width *W*, and height *H*) among a firm's vehicle fleet and among competitors' vehicles; R^2 is the coefficient of determination representing the proportion of variation in the data explained by the proposed model as applied in linear regression. We ignored product-attribute endogeneity, assuming that the observed nonprice attributes are uncorrelated with the unobserved utility component.

4.2 Specification. The utility specification can be broken into three parts following Eq. (1): terms that rely on the product alone $\delta^\top \mathbf{d}_j$, namely, brand dummies for European, Japanese, Chrysler, General Motors, and Korean vehicles; Ford was considered the baseline brand, so no dummy variable was used for Ford vehicles;

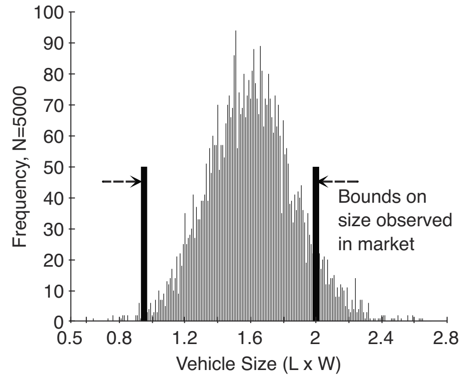

Fig. 1 Distribution of ideal vehicle size implied by footprint parameters in model 2

interactions between product attributes and demographics $\boldsymbol{\beta}^\top \mathbf{b}_{ij}$, namely, minivan and children $b_{m\cdot c}$, SUV and children $b_{s\cdot c}$, pickup truck and rural living $b_{p\cdot r}$; product-attribute or attribute-demographic interaction terms with individual-specific random coefficients assumed normally distributed $\boldsymbol{\mu}_i^\top \mathbf{m}_{ij}$, namely, vehicle price divided by income $p_j/s_{\text{inc},i}$, power to weight ratio (a proxy for acceleration) z_{hp}/z_{VM}, combined city and highway fuel consumption $100/z_{\text{MPG}}$, vehicle footprint $z_L z_W$; class dummies based on U.S. Environmental Protection Agency vehicle classes, namely, two-seater or minicompact, minivan, SUV, full-size van, pickup truck; and a hybrid powertrain dummy. Note that z_{hp}/z_{VM} is $z_{hp}\times 10/z_{VM}$ where z_{VM} is measured in lbm, and footprint is $z_L z_W$ measured in in.$^2/10{,}000$. It should be noted that the choice of interaction terms may affect results.

Two models were estimated. Model 1 assumes utility is monotonic in vehicle footprint: $\mu_1(z_L z_W)$. Model 2 assumes an ideal-point model of footprint: $\mu_1(\mu_2 - z_L z_W)^2$, which implies an interior maximum when μ_1 is negative. Variation across individuals in μ_2 represent individual-specific ideal footprints for a vehicle, capturing horizontally differentiated preferences. In order to use estimation techniques built around linear-in-parameter utilities, we simplified the expression by expanding the quadratic and dropping $\mu_1\mu_2^2$, leaving $\mu_1(z_L z_W)^2 - \hat{\mu}_2 2 z_L z_W$, where $\hat{\mu}_2=\mu_1\mu_2$, which we used in the estimation. We can drop $\mu_1\mu_2^2$ from the expression because it is constant across vehicles and only relative utility affects choice probabilities. We assume that μ_1 and $\hat{\mu}_2$ are independent and normally distributed resulting in different model behavior compared with a nonlinear model where μ_1 and μ_2 were assumed independent and normally distributed. Although maintaining different distributional properties than the typical quadratic ideal-point model, the model as estimated provides an effective ideal-point as shown by the simulated distribution of ideal-points in Fig. 1. Parameter estimates and standard errors are reported in Table 1.

4.3 Model Performance. We evaluate the performance of the two models according to the criteria described in Sec. 3.

4.3.1 Interpretability and Fit. Attributes and demographics were chosen with physical interpretations related to vehicle design. The price term allows sensitivity to price to change nonlinearly with income, following the intuition that price influences choices more when it represents a higher percentage of annual income. Horsepower z_{hp} over curbweight z_{VM} and vehicle size are also deemed relevant to the car-buying decision. However, the monotonic formulation of model 1 seems nonsensical when carried to extremes. The formulation in model 2 maps more naturally to the observation that different size vehicles (of the same price) succeed in the market.

Table 1 Vehicle demand model parameter estimates

Mean values				
	Model 1		Model 2	
Parameters	Estimate	Std. error	Estimate	Std. error
p/s_{inc}	−2.89	0.12	−3.83	0.15
$10z_{hp}/z_{VM}$	0.36	0.13	0.61	0.13
$100/z_{\text{MPG}}$	−0.84	0.028	−0.88	0.031
$z_L z_W$	5.03	0.12		
$(z_L z_W)^2$			−18.8	1.17
$-2z_L z_W$			−30.0	1.68
$m_{tw,mc}$	−1.69	0.42	−0.063[a]	0.27
m_{minivan}	−7.53	0.96	−4.74	0.58
m_{SUV}	−0.66	0.12	−0.61	0.11
m_{van}	−4.61	1.11	−4.49	0.50
m_{pickup}	−5.87	0.56	−1.59	0.13
v_{HEV}	−3.95	0.45	−3.27	0.34
d_{Europe}	−0.45	0.055	−0.25	0.058
d_{Japan}	0.11	0.036	0.19	0.038
d_{Chrysler}	0.13	0.043	0.11	0.044
d_{GM}	−0.43	0.036	−0.35	0.038
d_{Korea}	−0.58	0.058	−0.51	0.059
$b_{m\cdot c}$	2.93	0.42	2.08	0.27
$b_{s\cdot c}$	0.93	0.15	0.86	0.13
$b_{p\cdot r}$	6.36	0.61	2.26	0.21
Standard deviations				
p/s_{inc}	0.13[a]	0.25	0.52	0.23
$10z_{hp}/z_{VM}$	0.77	0.27	0.021[a]	0.35
$100/z_{\text{MPG}}$	0.76	0.026	1.03	0.032
$z_L z_W$	0.45[a]	0.26		
$(z_L z_W)^2$			1.27	0.36
$-2z_L z_W$			4.60	0.28
$m_{tw,mc}$	2.28	0.27	1.32	0.26
m_{minivan}	6.34	0.65	4.39	0.41
m_{SUV}	3.42	0.16	3.09	0.14
m_{van}	2.64	0.85	2.65	0.40
m_{pickup}	7.30	0.58	2.17	0.15
v_{HEV}	1.99	0.36	1.22	0.39

[a]Not significant at 95% confidence interval.

Segment dummies were included to account for preferences not captured through trade-offs of other observed attributes. A hybrid dummy was also included to account for any influences on purchase decisions of hybrids independent of fuel economy. Similarly, brand dummies were explicitly included to account for brand preferences independent of other attributes. The demographic interactions with vehicle segments help to capture some intuitive preference heterogeneities.

The signs of both sets of parameters are generally as expected including for the footprint terms in model 2, which imply that consumers have maximum preferred size in the range of currently available vehicles. Figure 1 is a histogram of ideal vehicle size $L^* = \beta_{-2z_L z_W}/\beta_{(z_L z_W)^2}$ computed from a sample of 5000 random draws from the joint distribution of $\beta_{(z_L z_W)^2}$ and $\beta_{-2z_L z_W}$. The plot shows that approximately 92% of individuals have an ideal size in the interior of current vehicle offering sizes.

It is nonintuitive that individuals would prefer lower fuel economy, but the large standard deviations on the fuel consumption parameter imply this behavior for some consumers. This is a case where either the model lacks appropriate instruments to tease apart the power-to-weight ratio versus fuel consumption interaction, the model should be respecified to get at a more intuitive preference structure, or the data set should be improved.

Model 2 includes two more terms than model 1 and has two more terms that are significant in a two-tailed t-test at a 95% confidence interval as noted in Table 1. Notably, the standard deviations for price and footprint in model 1 are not significant. Model 2 appears to capture additional consumer heterogeneity with both means and standard deviations of the footprint terms significant. However, the mean of the two-seater vehicle class and the standard deviation of horsepower to weight are not significant at the 95% confidence interval.

Regarding fit, the likelihood ratio index, or pseudo-R^2 value, for the models were 0.116 and 0.120, respectively. The likelihood ratio test resulted in values for $-2 \ln L^*$ that far exceeded the critical values at 99% confidence as shown in Table 2. Table 2 shows the results from the likelihood ratio test for the original estimation and for a pseudo-hold-out sample discussed in Sec. 4.3.2, where Ω_1 and Ω_2 represent all model parameters for models 1 and 2, respectively. The first two sets of data compare model 1 and model 2 to the case of equal shares for all products, or no model. The third set compares models 1 and 2 directly by recognizing that model 2 differs from model 1 by one additional random coefficient corresponding to the $(z_L z_W)^2$ term. Two parameters are associated with each normally distributed random coefficient (i.e., the mean and standard deviation), therefore two degrees of freedom are used in the chi-square distribution. For measures against no model and for the direct comparison, model 2 shows improved fit over model 1.

Table 2 Results for likelihood ratio test for model 1 and model 2 for the original estimation and for a PHO sample

Model	Hypothesis	Log-likelihood	$-2 \ln L^*$	Degrees of freedom	Critical value at 99% confidence
Null versus model 1	$LL(0)$	−47,542			
	$LL(\Omega_1)$	−42,007	11,071	28	48.3
Null versus model 2	$LL(0)$	−47,542			
	$LL(\Omega_2)$	−41,839	11,406	30	50.9
Model 1 versus model 2	$LL(\Omega_1)$	−42,007			
	$LL(\Omega_2)$	−41,839	335	2	9.2
PHO model 1	$LL(0)$	−47,682			
	$LL(\Omega_1)$	−42,250	10,865	28	48.3
PHO model 2	$LL(0)$	−47,682			
	$LL(\Omega_2)$	−42,184	10,998	30	50.9
PHO model 1 versus model 2	$L(\Omega_1)$	−42,250			
	$LL(\Omega_2)$	−42,184	133	2	9.2

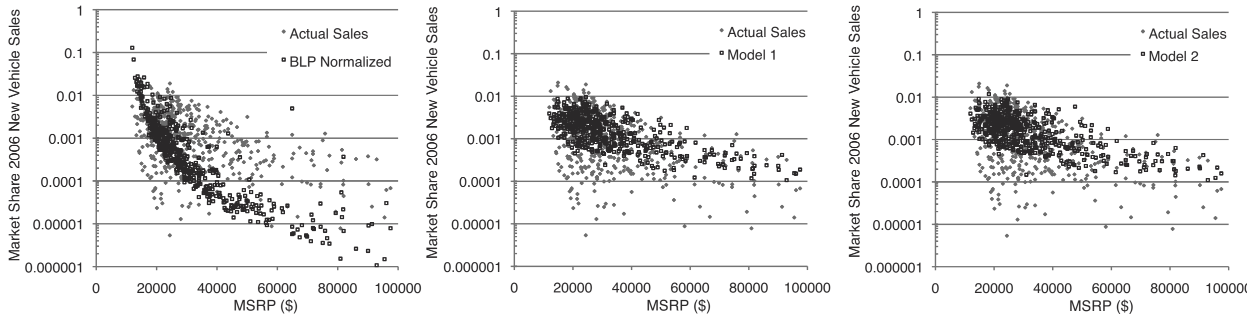

Fig. 2 Predicted shares versus actual shares. Left: BLP; center: model 1; right: model 2

4.3.2 Predictive Validity and Plausibility. The use of choice-based sampling did not allow true hold-out sample testing due to the small sample size for some vehicle models in the survey data. Instead, a psuedo-hold-out (PHO) sample was generated by drawing a new choice-based sample from the 2006 survey data. This new sample overlaps the original because individuals who purchased low-observation vehicle alternatives in the survey data are identical by necessity in both the original and hold-out samples; however, the overlap is small, only 84 out of 6563 individuals. A preferred approach would be to compose a hold-out sample from data for a subsequent year that would allow the model to be tested across individuals and vehicle alternatives.

The log-likelihood ratios for the PHO for models 1 and 2 were 0.114 and 0.115, respectively, indicating that the ideal-point model had slightly higher predictive validity than the monotonic model. The lower portion of Table 2 shows the results from the likelihood ratio test for the PHO sample, where Ω_1 and Ω_2 represent all model parameters for models 1 and 2, respectively. The first two sets of data compare models 1 and 2 to the case of no model. The third set compares models 1 and 2 directly by recognizing that model 2 differs from model 1 by one additional random coefficient corresponding to the $(z_L z_W)^2$ term. For both measures model 2 shows improved predictive validity over model 1.

To evaluate plausibility, we used three assessments to examine market shares and substitution patterns predicted by the models. Figure 2 plots predicted market shares from BLP, model 1, and model 2 with observed market shares. A conventional economics study would include alternative-specific constants to match market shares exactly. While this is important for producing accurate predictions of market shares, we emphasize the importance of first considering model performance without the aid of the alternative-specific constants because designers comparing different models should consider how much variance can be described by design and brand attributes before the aid of the constants.

We observe from Fig. 2 that models 1 and 2 underpredict choice shares for the most popular vehicle alternatives and overpredict choice shares for the least popular vehicle alternatives. We also observe that the BLP model applied to the 2006 vehicle data overpredicts the shares of the least expensive vehicles and underpredicts the shares of the most expensive vehicles.

The purpose for the comparison with the BLP model is not to compare performance directly per se given that BLP was estimated on 1971–1990 vehicle data, but to illustrate a potential pitfall in adopting "off the shelf" choice models in a design for market systems study. To make the comparison as fair as possible, all attributes involving dollar values were scaled from 2006 dollars to 1983 dollars for BLP utility evaluation. Additionally, a model of the outside good was reported in BLP but not in models 1 and 2. For BLP predicted market shares, we eliminated individuals whose maximum utility did not exceed the value of the outside good and then rescaled the market shares based on the remaining individuals.

For the second assessment, we examined the substitution patterns produced by both models by looking at own- and cross-elasticities. Elasticities are important for design studies because they indicate how changes in design attributes would affect demand in the local region of existing products. We simulated own- and cross-elasticities for each vehicle alternative using the estimation population of individuals ($I=6563$) and 100 standard normal random draws for each individual.

Comparing own-elasticities revealed that model 2 has greater heterogeneity in the $z_L z_W$ and gal/100 mi attributes. Another important difference is that while model 1 indicates that, on average, size is more important as the vehicle size increases (shown by increasing elasticities from smaller to larger classes), model 2 shows that increased size is more important for the large sedans and much less important for full-size vans and pickups. We also observed differences in cross-elasticities between models 1 and 2 but do not report these due to space limitations.

The third assessment considers the substitution pattern metric over the feasible design space of a midsize crossover vehicle based on an engineering design model described in Ref. [38]. Here, 10,000 points were sampled from the vehicle design space using LATIN HYPERCUBE sampling, and ten vehicle designs were chosen from the set of feasible vehicle designs by assigning a score to each feasible vehicle and then sampling at even increments across the range of scores. A vehicle was scored by taking the L2-norm of the normalized values (over the observed range) for the quantitative vehicle attributes z_{VM}/z_{hp}, $z_L z_W$, and mile/gal. The substitution pattern metric was computed for each of the ten designs. The predicted choice share for the original vehicle alternative in the original choice set and the combined choice share of the original and identical alternative in the augmented choice set as well as the mean values Ψ are reported for each demand model in Table 3.

Table 3 shows that Ψ is very low for all demand models. This implies that cross substitution from the original to the identical alternative in the augmented choice set is similar to IIA rather than perfect substitution. The values of Ψ' in Table 3 are similarly low implying that substitution patterns of the demand models achieve only a minor percentage change from proportional substitution toward perfect substitution. While substitution patterns for the demand models studied here do not follow the IIA property, the substitution patterns look more like proportional substitution rather than perfect substitution between identical alternatives.

These results bring into question the interpretability of the design study in Sec. 5 that relies on the implied substitution patterns

Table 3 Mean values across design space for substitution pattern metrics for each demand model

Mean values	Model 1	Model 2	BLP
Original share	0.00272	0.00285	0.00018
Combined new share	0.00539	0.00562	0.00037
Ψ	0.00840	0.0113	0.00186
Ψ'	0.0112	0.0167	0.00334

Table 4 Design optimization results: attributes and design variables

	z_{MPG}	z_{VM}	z_{hp}	v_{TG65} (%)	v_{TS}	v_{CVI}	Other active constraints	x_B	x_{BtS}	x_{FD}	x_{L103}	x_{L101}	x_{H101}	x_{W105}
Model 1	21.7	4230	216	5.0*	115*	46	v_{A107}, $v_{\min SH}$	83.6	0.95*	3.44	197	120*	67	79*
Model 2	21.7	4230	216	5.0*	115*	46	v_{A107}, $v_{\min SH}$	83.6	0.95*	3.44	197	120*	67	79*
BLP	18.5	3685	310	11.3	115*	15*	v_{A107}, $v_{\min SH}$, v_R	100*	1.18*	3.74	174	96	67	74

The * signifies an active constraint or variable bound.

of the demand model to inform product-attribute positioning for a new product given an existing choice set or the repositioning of product attributes in a fixed choice set. For example, the more that substitution between identical alternatives follows proportional substitution, the weaker price competition will be between identical alternatives. Thus, homogeneity of product attributes is "subsidized" relative to our expectation of the real market because the two identical vehicles compete head to head for only a small fraction of their mutual sales. Arguments can be made for why a demand model reflecting perfect substitution between implied identical alternatives would similarly fail to reflect real market behavior. However, it is difficult to imagine that an identical vehicle, even in an emerging segment and from a different carmaker, would attract only 0.2–1.1% of the market share of its twin as implied by the values of Ψ reported here.

The potential repercussions for design optimization are clear. A firm could increase its own market share (in the design study) by simply introducing an identical copy of an existing product if the demand model exhibits proportional substitution (even when IIA is overcome such as with a mixed-logit model). Similarly, a competing firm could introduce the identical product and rather than split the market share with the existing product gain additional market share from other products in the market. Such substitution patterns may be appropriate in some markets, but they violate our intuition about rational substitution where market share would be split evenly by two identical alternatives rather than a product gaining additional market share simply by duplicating itself.

5 Vehicle Design Example

We conduct design studies for the U.S. automotive market assuming one, two, or five firms are simultaneously designing a midsize crossover. We remove the seven midsize crossover vehicles actually in the 2006 market prior to executing each study. We assume a simple cost function and a model of competition as described in Ref. [38]. We repeated the studies for each demand specification, and we compare design variables in Table 4 and market share, firm profit, and the substitution pattern metric for each designed vehicle in Table 5.

5.1 Cost Model. Vehicle costs were represented as a static cost vector of variable costs for nondesigned vehicles. The cost vector is derived from an assumed relationship between market prices, dealer markups, and original equipment manufacturer (OEM) markups. The cost model for designed vehicles was derived as a regression equation on product characteristics where the dependent variable is the vector of prespecified costs for each vehicle.

We make the common economic assumption that, for a given firm, the price they charge in the market increases with product quality and that firms practice cost-minimizing behavior, i.e., they seek the minimum cost of inputs to produce an output of a given quality. Therefore, vehicle price and cost should both be monotonic with respect to product quality, taken to be increasing levels of measurable product characteristics.

We also assume that there is a consistent relationship (or apportioning) between the dealer invoice and the amortized per vehicle OEM's cost and the dealer invoice and manufacturer's suggested retail price (MSRP) markup.

5.2 Market Competition. We assume a subgame perfect model of equilibrium where each firm designs its vehicle assuming that all other firms will update their product prices in response to the new design [39]. We used the relaxation approach to solve the top level (design) equilibrium problem [40] and a fixed point iteration method developed in Ref. [41] to solve the inner price equilibrium problem for the entire market at each upper level design optimization iteration. The relaxation method exhibited stable properties in our example but in general is not guaranteed to converge to market equilibrium.

Multiple firms $F-q+1,\ldots,F$ with competing vehicles in the same vehicle segment are designated as the designing firms. The designing firm f maximizes profit π_f with respect to the vehicle design variables $\mathbf{x}_{j,f}$ where all vehicle prices in the entire market $\mathbf{p}$ are determined by a pricing subgame.

Do while $\max\{\Delta\mathbf{z}_{j,f};f=F-q+1,\ldots,F\} > z_{\text{tol}}$

for $f=F-q+1,\ldots,F$

$$\max_{\mathbf{x}_{j,f},\mathbf{p}} \boldsymbol{\pi}_f(\mathbf{x}_{j,f},\mathbf{p};\mathbf{z}_{k\neq j}) = \sum_{j=1}^{J_f} Q_j(p_j - c_{v,j}) \tag{5}$$

s.t. $\mathbf{g}(\mathbf{x}_{j,f}) \le 0$

$$\mathbf{p} = \arg(\max_{\mathbf{p}}\{\pi_f(\mathbf{p}_f;\mathbf{z}_f);\ \ f=1,\ldots,F\})$$

end

end.

5.3 Optimization Results. The optimization problem is solved as described in Eq. (5) where each designing firm attempts to maximize profit for a portfolio of vehicles with respect to design variables for the designed vehicle and subject to cargo volume v_{CVI}, rollover score v_R, maximum grade at 65 mph while towing v_{TG65}, curb clearance geometry v_{A107}, minimum sitting height $v_{\min SH}$, and minimum top speed v_{TS} constraints and subject to the entire market being in price equilibrium. The design variables are engine bore x_B, engine bore to stroke ratio x_{BtS}, final drive ratio x_{FD}, vehicle length x_{L103}, vehicle wheelbase x_{L101}, vehicle height x_{H101}, and vehicle width x_{W105}. We present optimization results for each demand model and for design scenarios with one, two, and five designing firms in Tables 4 and 5. All designing firms produced the same vehicle design for a given demand model and for any number of designing firms. Firms produced the identical vehicle under models 1 and 2, but the vehicles were offered at different prices and achieved different market shares.

Design optimization results show that for particular assumptions about product cost, competition, and product constraints there is a single optimal vehicle design in each case. We observe that the values of the product attributes are bounded by constraints imposed to represent a particular vehicle segment. We then relaxed the upper bounds on the length, width, and wheelbase variables and found that there is a unique optimal vehicle design for model 1 different from model 2. However, both vehicle designs

Table 5 Design optimization results: price, shares, profit, and cost

Model 1	Price	P_j	Ψ	π_f	c_v
One firm	61,937	0.0081		56.7	23,657
Two firms	61,816	0.0080		56.3	—
	50,816	0.0062	0.019	9.6	—
Five firms	61,134	0.0074		54.3	—
	50,592	0.0057		9.1	—
	51,217	0.0111		16.5	—
	54,129	0.0098		27.7	—
	58,043	0.0084		19.3	26,344
Model 2	Price	P_j	Ψ	π_f	c_v
One firm	53,390	0.0118		45.2	23,657
Two firms	52,859	0.0113		44.6	—
	43,287	0.0097	0.047	7.8	—
Five firms	51,420	0.0097		14.0	—
	42,669	0.0079		7.1	—
	43,921	0.0147		13.3	—
	46,562	0.0126		22.0	—
	49,505	0.0107		14.0	26,344
BLP	Price	P_j	Ψ	π_f	c_v
One firm	29,770	0.0008		25.2	21,847
Two firms	29,765	0.0008		25.2	—
	27,486	0.0015	0.0057	7.4	—
Five firms	29,751	0.0008		25.0	—
	27,474	0.0014		7.3	—
	27,269	0.0015		3.5	—
	28,227	0.0012		6.9	—
	30,514	0.0007		2.3	24,162

The line — indicates that the value for this table entry is identical to the entry immediately above.

are much larger than the initial design space with the optimal design for model 1 at the boundary of vehicle size observed in the market.

All design solutions are constraint-bound solutions. Each case has seven degrees of freedom and seven active constraints or variable bounds. This indicates that the relaxed optimal vehicle design based on the firm's portfolio profit maximization objective would be outside the region defined a priori as a midsize crossover. With models 1 and 2, the optimal design is larger with more emphasis on fuel economy than fast 0–60 mph time. With the BLP model the optimal design is smaller with fast 0–60 mph time and low emphasis on fuel economy. Clearly, the choice of demand model has implications for the optimal vehicle design as well as the expected market share and firm profit with large differences computed between model 1, model 2, and BLP. The differences in price across firms come from differences in product portfolios and differences in brand valuation in models 1 and 2 and from differences in product portfolios in BLP.

The market equilibrium prices are quite high compared with the 2006 market prices under the models 1 and 2. High predicted prices were systematic across the entire market not only for the designed vehicles. One reason for this could be that models 1 and 2 underestimate price sensitivity since estimating is done without an outside good. Additionally, omitting alternative-specific constants to increase the demand of the most popular models and decrease the demand for the least popular models could similarly influence the estimate of price sensitivity. The fifth designed vehicle in the five-firm market case has a unit cost greater than the other four vehicles because it is an all-wheel drive vehicle.

The substitution pattern metric Ψ is computed comparing the two firm cases to the one firm case. The values of Ψ show a small percentage increase from proportional substitution similar to the values computed in the static cases shown in Table 3. Firms 1 and 2 show lower prices for the same vehicles in subsequent scenarios with additional designing firms. This is a result of price competition; however, the price reduction is modest. The small cross substitution between identical alternatives and the minimal price competition are two indicators that the proposed demand models are not suitable consumer representations for use in a practical design study, at least for the midsize crossover segment studied here.

6 Conclusion

We reviewed methods from the social sciences for evaluating the use of consumer choice models and interpreted those methods in a design study context. We used these methods to evaluate two forms of an automotive new vehicle choice model, respectively, emphasizing vertically and horizontally differentiated preferences for size. We found that allowing horizontal size preferences to be expressed explicitly and separately from distributions of random coefficients for vehicle class improves the performance of the model with respect to fit and predictive validity. With respect to plausibility, we showed that the inclusion of horizontal preference terms modifies substitution effects in terms of the elasticities and the substitution metric Ψ. We then observed differences in price and share prediction in design optimization results. We observed different optimal designs resulting between models 1 and 2 when we relaxed the variable bounds on length, wheelbase, and width.

We raise the concern that none of the demand models we tested were adequate for the particular optimization problem studied. Predictive validity and fit should be much higher. The substitution behavior appears unrealistic for comparing similar product offerings in a design optimization study for midsize crossover vehicles. Using the proposed substitution pattern metric, we observe that all models tend toward proportional substitution rather than perfect substitution between identical alternatives. The implication for interpreting design studies that employ mixed-logit demand models is that there will be an artificial incentive toward design homogeneity if a given market actually behaves closer to perfect substitution for identical alternatives rather than proportional substitution. This issue is potentially compounded for other logit-based models such as nested-logit or latent class because these models assume larger aggregations of consumers behaving according to the IIA substitution.

Future work should explore the question of the adequacy of the mixed-logit model to represent substitution behavior for design studies in general. Getting the substitution patterns wrong would prevent achieving many of the stated objectives of design for market systems research, including studying market entry and exit, product-attribute competition among firms, product portfolio optimization, and estimation of cannibalization effects.

Applying similar evaluation methods to those presented should be foundational to DBD research that includes models of consumer choice. Similar evaluation methods are needed for producer cost models and for game-theoretic competitive behavior. The right combination of appropriate demand and cost models, and competitive assumptions, should lead to intuitive checks on plausibility applied to the entire market system.

While this article presented methods that are useful for comparing one demand model specification to another, it does not address the issue of how good is "good enough." Practically speaking, a good enough specification is one that yields sufficient predictive validity for the design context of interest. In principle, a good enough specification will embody economic and behavioral theory that can be tested in the context of the particular market under consideration.

Many elements of a market system, not only consumer taste differences, influence the product differentiation observed in the market, including perception of brand, aesthetic qualities, distribution networks and product availability, firm's cost structure, and firm's attainable technology set. All of these factors are likely important, yet none of them replaces the role of consumer taste differences in its relationship to differentiated-product design. Exploring the type of substitution effects that are reasonable for given markets and seeking demand models with the appropriate properties appears to be an immediate challenge of design for market systems research. The demand modeling approach should be validated before design optimization studies with market system objectives can be usefully interpreted and implemented.

Acknowledgment

The authors would like to thank W.R. Morrow and E. Suel for their contributions to the utility evaluation code, F. Feinberg for the helpful comments on the demand model evaluations, K. Bolon for the assistance in collecting and coordinating data sets, and to Maritz Holdings Inc. for the use of their data. We gratefully acknowledge AVL Advanced Simulation Technologies for providing a license to AVL Cruise used in the development of the vehicle simulation. This work has been supported by the University of Michigan-Ford Motor Co. Innovation Alliance, the Rackham Graduate School of the University of Michigan, and the National Science Foundation under Grant No. DMI-0503737. This support is gratefully acknowledged. The opinions expressed are only those of the authors.

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